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Pseudo-Dirac Neutrinos, a Challenge for Neutrino Telescopes

John F. Beacom,[1] Nicole F. Bell,[1,2] Dan Hooper,[3]
John G. Learned,[4,2] Sandip Pakvasa,[4,2] and Thomas J. Weiler[5,2]

[1] *NASA/Fermilab Astrophysics Center, Fermi National Accelerator Laboratory, Batavia, Illinois 60510-0500*
[2] *Kavli Institute for Theoretical Physics, University of California, Santa Barbara 93106*
[3] *Department of Physics, University of Wisconsin, Madison, Wisconsin 53706*
[4] *Department of Physics and Astronomy, University of Hawaii, Honolulu, Hawaii 96822*
[5] *Department of Physics and Astronomy, Vanderbilt University, Nashville, Tennessee 37235*

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Neutrinos may be pseudo-Dirac states, such that each generation is actually composed of two maximally-mixed Majorana neutrinos separated by a tiny mass difference. The usual active neutrino oscillation phenomenology would be unaltered if the pseudo-Dirac splittings are $\delta m^2 \lesssim 10^{-12}$ eV^2; in addition, neutrinoless double beta decay would be highly suppressed. However, it may be possible to distinguish pseudo-Dirac from Dirac neutrinos using high-energy astrophysical neutrinos. By measuring flavor ratios as a function of L/E, mass-squared differences down to $\delta m^2 \sim 10^{-18}$ eV^2 can be reached. We comment on the possibility of probing cosmological parameters with neutrinos.

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Are neutrinos Dirac or Majorana fermions? Despite the enormous strides made in neutrino physics over the last few years, this most fundamental and difficult of questions remains unanswered. The observation of neutrinoless double beta decay would unambiguously signal Majorana mass terms and hence lepton number violation. If no positive signal from neutrinoless double beta decay is seen, it may be tempting to conclude that neutrinos are Dirac particles, particularly if there is independent evidence from tritium beta decay or cosmology for significant neutrino masses. However, Majorana mass terms (and lepton flavor violation) may still exist, though their effects would be hidden from most experiments. Observations with neutrino telescopes may be the only way to reveal their existence.

The generic mass matrix in the $(\nu_L, (\nu_R)^C)$ basis is

$$\begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix}. \tag{1}$$

A Dirac neutrino corresponds to the case where $m_L = m_R = 0$, and may be thought of as the limit of two degenerate Majorana neutrinos with opposite CP parity. Alternatively, we may form a pseudo-Dirac neutrino [1, 2] by the addition of tiny Majorana mass terms $m_L, m_R \ll m_D$, which have the effect of splitting the Dirac neutrino into a pair of almost degenerate Majorana neutrinos, each with mass $\sim m_D$. The mixing angle between the active and sterile states is very close to maximal, $\tan(2\theta) = 2m_D/(m_R - m_L) \gg 1$, and the mass-squared difference is $\delta m^2 \simeq 2m_D(m_L + m_R)$. For three generations, the mass spectrum is shown in Fig. 1. The mirror model can produce a very similar mass spectrum [3].

The current theoretical prejudice is for the right-handed Majorana mass term to be very large, $m_R \gg m_D$, giving rise to the see-saw mechanism. Then the right-handed states are effectively hidden from low energy phenomenology, since their mixing with the active states is suppressed through tiny mixing angles. This is desirable, since no direct evidence for right-handed (sterile) states has been observed (we treat both solar and atmospheric neutrinos as active-active transitions, and do not attempt to explain the LSND [4] anomaly).

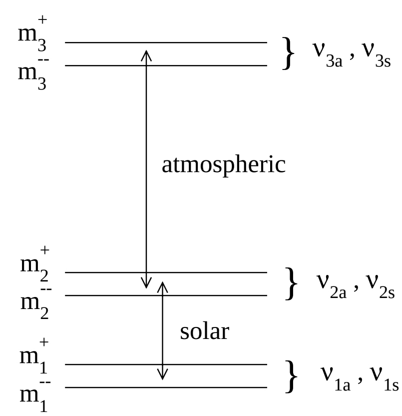

FIG. 1: The neutrino mass spectrum, showing the usual solar and atmospheric mass differences, as well as the pseudo-Dirac splittings in each generation (though shown as equal, we assume they are independent). The active and sterile components of each pseudo-Dirac pair are ν_{ja} and ν_{js}, and are maximal mixtures of the mass eigenstates ν_j^+ and ν_j^-. Neither the ordering of the active neutrino hierarchy, nor the signs of the pseudo-Dirac splittings, has any effect on our discussion.

If right-handed neutrinos exist, where else can they hide? The only alternative to the see-saw mechanism is pseudo-Dirac neutrinos. Here, although the mixing between active and sterile states is maximal, such neutrinos will, in most cases, be indistinguishable from Dirac neutrinos, as very few experiments can probe very tiny mass-squared differences.

In the Standard Model, m_D arises from the conventional Yukawa couplings and hence its scale is comparable to other fermion masses. In the see-saw model, m_R is identified with some large GUT or intermediate scale mass, and thus small neutrino masses are achieved. For pseudo-Dirac masses, on the other hand, we need both m_L and m_R to be small compared to m_D. The smallness of m_L with respect to m_D follows from their $SU(2)_L$ properties; the former breaks it while the latter is invariant under it. This suggests that a similar property with respect to a $SU(2)_R$ can also make m_R small compared to m_D. This can be obtained with an appropriate low-energy $SU(2)_L \otimes SU(2)_R$ symmetry group. Specific examples which achieve precisely this are given in Ref. [5]. There still remains the problem of keeping m_D itself small enough to keep the physical neutrino masses small compared to other fermions. There are a number of suggestions of how this may arise. One proposal is based on Dirac masses generated from higher-dimensional operators [6]. Other possibilities are: (i) Dirac masses are small because they are forbidden at tree level by a discrete symmetry and arise only as a result of a "Dirac see-saw" [7] or (ii) to invoke extra dimensions [8].

To uncover oscillation effects, a baseline of length $L \gtrsim E/\delta m^2$ is required. Astronomical-scale baselines will clearly be a powerful tool for probing very tiny δm^2 [9]. Crocker, Melia, and Volkas have considered possible distortions to the ν_μ spectrum [10].

Fig. 2 shows the range of neutrino mass-squared differences that can be probed with different classes of experiments. Present limits on pseudo-Dirac splittings arise from the solar and atmospheric neutrino measurements. Splittings of less than about 10^{-12} eV^2 (for ν_1 and ν_2) have no effect on the solar neutrino flux, while a pseudo-Dirac splitting of ν_3 could be as large as about 10^{-4} eV^2 before it would affect the atmospheric neutrinos.

Note that models with light sterile neutrinos often conflict with big bang nucleosynthesis limits on the number of light degrees of freedom in thermal equilibrium in the early universe. However, the sterile component of each Pseudo-Dirac pair will not be populated, provided the mass splitting of each pair is sufficiently small, as will be the case for the examples we consider here.

Formalism.— Let $(\nu_1^+, \nu_2^+, \nu_3^+; \nu_1^-, \nu_2^-, \nu_3^-)$ denote the six mass eigenstates, where ν^+ and ν^- are a nearly-degenerate pair. A 6×6 mixing matrix rotates the mass basis into the flavor basis $(\nu_e, \nu_\mu, \nu_\tau; \nu_e', \nu_\mu', \nu_\tau')$. In general, for six Majorana neutrinos, there would be fifteen rotation angles and fifteen phases. However, for pseudo-Dirac neutrinos, Kobayashi and Lim [2] have given an elegant proof that the 6×6 matrix $V_{\rm KL}$ takes the very simple form (to lowest order in $\delta m^2/m^2$):

$$V_{\rm KL} = \begin{pmatrix} U & 0 \\ 0 & U_R \end{pmatrix} \cdot \begin{pmatrix} V_1 & iV_1 \\ V_2 & -iV_2 \end{pmatrix}, \tag{2}$$

where the 3×3 matrix U is just the usual mixing matrix

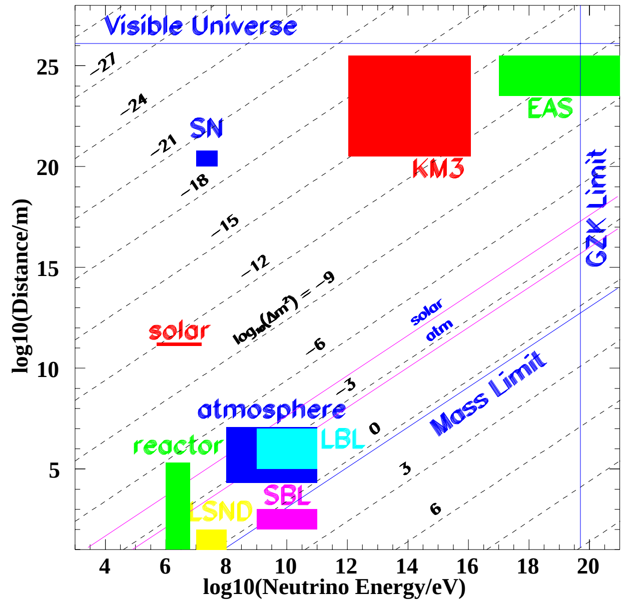

FIG. 2: The ranges of distance and energy covered in various neutrino experiments. The diagonal lines indicate the mass-squared differences (in eV^2) that can be probed with vacuum oscillations; at a given L/E, larger δm^2 values can be probed by averaged-out oscillations. We focus on a neutrino telescope of 1-km scale (denoted "KM3"), or larger, if necessary.

determined by the atmospheric and solar observations, the 3×3 matrix U_R is an unknown unitary matrix, and V_1 and V_2 are the diagonal matrices $V_1 = {\rm diag}(1,1,1)/\sqrt{2}$, and $V_2 = {\rm diag}(e^{-i\phi_1}, e^{-i\phi_2}, e^{-i\phi_3})/\sqrt{2}$, with the ϕ_i being arbitrary phases.

As a result, the three active neutrino states are described in terms of the six mass eigenstates as:

$$\nu_{\alpha L} = U_{\alpha j}\frac{1}{\sqrt{2}}\left(\nu_j^+ + i\nu_j^-\right). \tag{3}$$

The nontrivial matrices U_R and V_2 are not accessible to active flavor measurements. The flavor conversion probability can thus be expressed as

$$P_{\alpha\beta} = \frac{1}{4}\left|\sum_{j=1}^{3} U_{\alpha j}\left\{e^{i(m_j^+)^2 L/2E} + e^{i(m_j^-)^2 L/2E}\right\} U_{\beta j}^*\right|^2. \tag{4}$$

The flavor-conserving probability is also given by this formula, with $\beta = \alpha$. Hence, in the description of the three active neutrinos, the only new parameters beyond the usual three angles and one phase are the three pseudo-Dirac mass differences, $\delta m_j^2 \equiv (m_j^+)^2 - (m_j^-)^2$. In the limit that the δm_j^2 are negligible, the oscillation formulas reduce to the standard ones and there is no way to discern the pseudo-Dirac nature of the neutrinos.

We assume throughout that the neutrinos oscillate in vacuum. Lunardini and Smirnov have shown that the matter potential from relic neutrinos can affect the vacuum oscillation probabilities of astrophysical neutrinos, but only if the neutrino-antineutrino asymmetry of the background is large, of order 1 [11]. For present limits on that asymmetry, of order 0.1 [12], or for less extreme redshifts than they assume, matter effects are negligible [11].

Supernova neutrinos from distances exceeding $(E/10\ \mathrm{MeV})(10^{-15}\ \mathrm{eV}^2/\delta m^2)$ parsecs will arrive as a 50/50 mixture of active and sterile neutrinos due to vacuum oscillations. However, we focus on the potentially cleaner signature of flavor ratios of high-energy astrophysical neutrinos.

***L/E*-Dependent Flavor Ratios.**— Given the enormous pathlength between astrophysical neutrino sources and Earth, the phases due to the relatively large solar and atmospheric mass-squared differences will average out (or equivalently, decohere). The neutrino density matrix ρ is then mixed with respect to the three usual mass states but coherent between the two components of each pseudo-Dirac pair:

$$\rho \;=\; \frac{1}{2}\sum_{\alpha} w_\alpha \sum_{j=1}^{3} |U_{\alpha j}|^2 \Big\{ |\nu_j^+\rangle\langle\nu_j^+| \,+\, |\nu_j^-\rangle\langle\nu_j^-| \qquad (5)$$
$$+\; i e^{-i\delta m_j^2 L/2E}\, |\nu_j^-\rangle\langle\nu_j^+| - i e^{+i\delta m_j^2 L/2E}\, |\nu_j^+\rangle\langle\nu_j^-| \Big\}$$

Here w_α is the relative flux of ν_α at the source, such that $\sum_\alpha w_\alpha = 1$. The probability for a neutrino telescope to measure flavor ν_β is then $P_\beta = \langle\nu_\beta|\rho|\nu_\beta\rangle$, which becomes

$$P_\beta = \sum_\alpha w_\alpha \sum_{j=1}^{3} |U_{\alpha j}|^2\, |U_{\beta j}|^2 \left[1 - \sin^2\left(\frac{\delta m_j^2\, L}{4E}\right)\right] . \qquad (6)$$

In the limit that $\delta m_j^2 \to 0$, Eq. (6) reproduces the standard expressions. The new oscillation terms are negligible until E/L becomes as small as the tiny pseudo-Dirac mass-squared splittings δm_j^2.

Since $|U_{e3}|^2 \simeq 0$, the mixing matrix U for three active neutrinos is well approximated by the product of two rotations, described by the "solar angle" θ_{solar} and the "atmospheric angle" $\theta_{\mathrm{atm}} \simeq 45^\circ$. The pion production and decay chain at the source produces expected fluxes of $w_e = 1/3$ and $w_\mu = 2/3$. In the absence of pseudo-Dirac splittings, it is well known [13] that this results in $P_\beta \simeq 1/3$ for all flavors. Specifically, the detected flavor ratios are $\nu_e : \nu_\mu : \nu_\tau = 1 : 1 : 1$. Here and elsewhere, this $\nu_\mu - \nu_\tau$ symmetry obtains when $\theta_{\mathrm{atm}} = 45^\circ$ and $U_{e3} = 0$. If pseudo-Dirac splittings are present, we thus expect

$$\delta P_\beta \;\equiv\; -\frac{1}{3}\left[|U_{\beta 1}|^2\, \chi_1 + |U_{\beta 2}|^2\, \chi_2 + |U_{\beta 3}|^2\, \chi_3\right] , \qquad (7)$$

where $\delta P_\beta \equiv P_\beta - \frac{1}{3}$, and we have defined, for shorthand,

$$\chi_j \equiv \sin^2\left(\frac{\delta m_j^2\, L}{4E}\right) . \qquad (8)$$

TABLE I: Flavor ratios $\nu_e : \nu_\mu$ for various scenarios. The numbers j under the arrows denote the pseudo-Dirac splittings, δm_j^2, which become accessible as L/E increases. Oscillation averaging is assumed after each transition j. We have used $\theta_{\mathrm{atm}} = 45^\circ$, $\theta_{\mathrm{solar}} = 30^\circ$, and $U_{e3} = 0$.

1 : 1	$\xrightarrow[3]{}$	4/3 : 1	$\xrightarrow[2,3]{}$	14/9 : 1	$\xrightarrow[1,2,3]{}$	1 : 1
1 : 1	$\xrightarrow[1]{}$	2/3 : 1	$\xrightarrow[1,2]{}$	2/3 : 1	$\xrightarrow[1,2,3]{}$	1 : 1
1 : 1	$\xrightarrow[2]{}$	14/13 : 1	$\xrightarrow[2,3]{}$	14/9 : 1	$\xrightarrow[1,2,3]{}$	1 : 1
1 : 1	$\xrightarrow[1]{}$	2/3 : 1	$\xrightarrow[1,3]{}$	10/11 : 1	$\xrightarrow[1,2,3]{}$	1 : 1
1 : 1	$\xrightarrow[3]{}$	4/3 : 1	$\xrightarrow[1,3]{}$	10/11 : 1	$\xrightarrow[1,2,3]{}$	1 : 1
1 : 1	$\xrightarrow[2]{}$	14/13 : 1	$\xrightarrow[1,2]{}$	2/3 : 1	$\xrightarrow[1,2,3]{}$	1 : 1

In the absence of pseudo-Dirac terms, flavor democracy is expected. However, the pseudo-Dirac splittings lead to an oscillatory, flavor-dependent, reduction in flux, allowing us to test the possible pseudo-Dirac nature of the neutrinos with neutrino telescopes. The signatures are flavor ratios which depend on astronomically large L/E.

As a representative value, we take $\theta_{\mathrm{solar}} = 30^\circ$. Then the flavors deviate from the democratic $\frac{1}{3}$ value by

$$\begin{aligned} \delta P_e \;&=\; -\frac{1}{3}\left[\frac{3}{4}\chi_1 + \frac{1}{4}\chi_2\right] , \\ \delta P_\mu = \delta P_\tau \;&=\; -\frac{1}{3}\left[\frac{1}{8}\chi_1 + \frac{3}{8}\chi_2 + \frac{1}{2}\chi_3\right] . \end{aligned} \qquad (9)$$

The latter equality is due to the $\nu_\mu - \nu_\tau$ symmetry.

We show in Table I how the $\nu_e : \nu_\mu$ ratio is altered if we cross the threshold for one, two, or all three of the pseudo-Dirac oscillations. The flavor ratios deviate from 1 : 1 when one or two of the pseudo-Dirac oscillation modes is accessible. In the ultimate limit where L/E is so large that all three oscillating factors have averaged to $\frac{1}{2}$, the flavor ratios return to 1 : 1, with only a net suppression of the measurable flux, by a factor of 1/2.

It was recently pointed out that astrophysical neutrino flavor ratios will deviate significantly from 1:1:1 if one or two of the active neutrino mass-eigenstates decay [14]. The result for that case bears some resemblance to the result presented here. In particular, if there is a range of L/E values where the one or two heavier mass states have oscillated with their pseudo-Dirac partners, but the light state has not, then a fraction $\frac{1}{2}$ of the heavy states will have disappeared, to be compared with the complete disappearance expected from unstable neutrinos [14]. The effects from the pseudo-Dirac mass differences are much milder and will require more accurate flavor measurements than for the decays [14, 15]. In addition, the active-active mixing angles will need to be known independently.

Neutrinoless Double Beta Decay.— Since the two mass eigenstates in each pseudo-Dirac pair have opposite

CP parity, no observable neutrinoless double beta decay rate is expected. The effective mass for neutrinoless double beta decay experiments is given by

$$\langle m\rangle_{\text{eff}} = \frac{1}{2}\sum_j U_{ej}^2\left(m_j^+ - m_j^-\right) = \frac{1}{2}\sum_j U_{ej}^2\frac{\delta m_j^2}{2m_j}, \quad (10)$$

which is unmeasurably small, $\langle m\rangle_{\text{eff}} \lesssim 10^{-4}$ eV for the inverted hierarchy and even less for the normal hierarchy. In contrast, in the mirror model [3], the sum above has $\left(m_j^+ + m_j^-\right)$, and can thus produce an observable signal.

Cosmology with Neutrinos.— It is fascinating that non-averaged oscillation phases, $\delta\phi_j = \delta m_j^2 t/4p$, and hence the factors χ_j, are rich in cosmological information [9]. Integrating the phase backwards in propagation time, with the momentum blue-shifted, one obtains

$$\begin{aligned}\delta\phi_j &= \int_0^{z_e} dz\frac{dt}{dz}\frac{\delta m_j^2}{4p_0(1+z)} \qquad (11)\\ &= \left(\frac{\delta m_j^2 H_0^{-1}}{4p_0}\right)\int_1^{1+z_e}\frac{d\omega}{\omega^2}\frac{1}{\sqrt{\omega^3\,\Omega_m + (1-\Omega_m)}},\end{aligned}$$

where z_e is the red-shift of the emitting source, and H_0^{-1} is the Hubble time, known to 10% [16]. This result holds for a flat universe, where $\Omega_m + \Omega_\Lambda = 1$. The integral is the fraction of the Hubble time available for neutrino transit and oscillation. For the presently preferred values $\Omega_m = 0.3$ and $\Omega_\Lambda = 0.7$, the asymptotic ($z_e \to \infty$) value of the integral is 0.53. This limit is approached rapidly: at $z_e = 1\,(2)$ the integral is already 77% (91%) saturated. For cosmologically distant ($z_e \gtrsim 1$) sources such as gamma-ray bursts, we may assume the asymptotic value with only $\sim 10\%$ error. A small set of non-averaged neutrino oscillation data would then allow one to deduce δm^2 to about 20%, without even knowing the source red-shifts. A larger data set with known z_e's from spectral measurements might allow one to estimate the values of Ω_M and Ω_Λ. Alternatively, known values of Ω_M and Ω_Λ would allow one to infer the source redshift z_e.

Such a scenario would be the first measurement of a cosmological parameter with particles other than photons. An advantage of measuring cosmological parameters with neutrinos is the fact that flavor mixing is a microscopic phenomena and hence presumably free of ambiguities such as source evolution or standard candle assumptions [9, 17]. Another method of measuring cosmological parameters with neutrinos is given in Ref. [18].

Conclusions.— Neutrino telescope measurements of neutrino flavor ratios may achieve a sensitivity to mass-squared differences as small as 10^{-18} eV^2. This can be used to probe possible tiny pseduo-Dirac splittings of each generation, and thus reveal Majorana mass terms (and lepton number violation) not discernable via any other means.

Note added: As this work was being finalized, a paper by Keranen *et al.* [19] appeared which addresses some of the issues herein.

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